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THEORETICAL
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Shielding of a Low-Frequency Electric Field by a Multilayer Circular Disk

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Abstract—We have solved the boundary-value problem with nonclassical boundary conditions, which describes the penetration of a low-frequency field through a multilayer thin circular disk. The solution of the problem is reduced to solving the Fredholm integral equation of the second kind. The influence of some parameters of the problem on the shielding factor is investigated numerically.

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INTRODUCTION

Sources of electromagnetic fields (EMFs) are radio and telecommunication devices, conventional and cell communication systems, and radio telephones. The level of the fields produced by technical activity of people is quite high. This deteriorates the parameters of electromagnetic surroundings and lowers the effectiveness of operation of technical equipment and personnel [1–3]. Low-frequency fields produce the strongest effect on human beings because biological functioning is based predominantly on low-frequency processes. Active and passive shielding of the space in which the level of electromagnetic field should be lowered or of technical devices which should be protected from the action of the external electromagnetic field is the most effective method of lowering the EMF level (electric and magnetic field strengths) in the surrounding medium [4–7]. A method for calculating low-frequency electromagnetic fields was proposed in [4, 8, 9] for the case when open screens are perfectly conducting. In this case, the field does not penetrate through the screen walls. However, for screens with a low conductivity of the material, the field penetrates through the shell wall, and such processes are simulated using nonclassical boundary conditions [10, 11].

1. FORMULATION OF THE PROBLEM

Suppose that a planar screen in the form of a circular disk D of radius a and thickness Δ is located in a homogeneous isotropic space R^3 with permittivity and permeability ε_0 and μ_0 (Fig. 1):

$$D = \left\{-\frac{\Delta}{2} < z < \frac{\Delta}{2},\ 0 \le \rho \le a,\ 0 \le \varphi \le 2\pi\right\}.$$

Disk D consists of plane-parallel layers

$$S_s = \{z_s < z < z_{s+1},\ 0 \le \rho \le a,\ 0 \le \varphi \le 2\pi\},$$
$$s = 1, \ldots, n,$$

where $z_1 = -\Delta/2$, $z_{n+1} = \Delta/2$, $\Delta_s = z_{s+1} - s_s$ is the thickness of the sth layer, and $\Delta = \sum_{s=1}^{n} \Delta_s$.

Layer S_s is made of a material with complex electromagnetic parameters ε_s and μ_s.

Electromagnetic field $\mathbf{E}^{(s)}$, $\mathbf{H}^{(s)}$ in layer S_s is described by the Maxwell equations $\operatorname{curl}\mathbf{E}^{(s)} = i\omega\mu_s\mathbf{H}^{(s)}$ and $\operatorname{curl}\mathbf{H}^{(s)} = -i\omega\varepsilon_s\mathbf{E}^{(s)}$, where ω is the cyclic frequency of the field.

At point O (center of disk D), we introduce cylindrical coordinates $\{\rho, \varphi, z\}$ and divide space R^3 by plane $\Gamma(z = 0)$, $\Gamma = \Gamma_a \cup \Gamma_0$, $\Gamma_0 = \{z = 0, \rho \ge a, 0 \le \varphi \le 2\pi\}$, and $\Gamma_a = \{z = 0, 0 \le \rho < a, 0 \le \varphi \le 2\pi\}$, into half-spaces $D_1(z < 0)$ and $D_2(z > 0)$. Disk D is bounded by circular planes $\Gamma^{\pm} = \{z = \pm\Delta/2, 0 \le \rho < a, 0 \le \varphi \le 2\pi\}$.

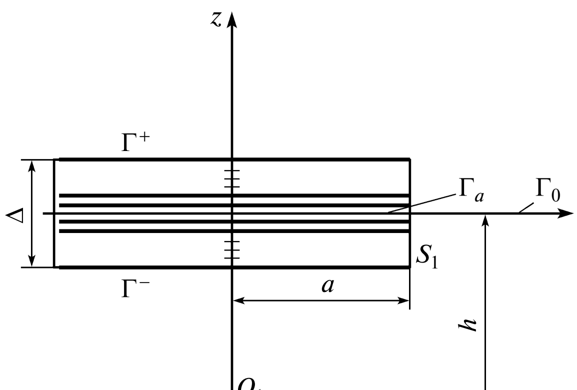


Fig. 1. Geometry of the problem.

A source of the low-frequency electric field (dipole) is located in half-space D_1 at point $O_1(0, 0, -h)$, $h > \Delta$. Let u_0 be the electric potential of the dipole, $u_j = u_0 + u_j'$ be the total electric field potential in domain D_j, $j = 1, 2$, and u_j' be the potential of the secondary electric field.

Let us formulate the boundary-value problem of shielding with special boundary conditions on the surface of disk D, simulating the penetration of the electric field through the screen.

We must determine secondary potentials u_j', $j = 1, 2$, which satisfy the Laplace equation $\Delta u_j' = 0$ and the boundary conditions

$$(u_1 - u_2)|_{\Gamma_0} = 0, \quad \frac{\partial}{\partial z}(u_1 - u_2)|_{\Gamma_0} = 0, \tag{1}$$

$$u_2|_{\Gamma^+} = u_1|_{\Gamma^-} + V, \tag{2}$$

$$\frac{\partial}{\partial z}u_2\Big|_{\Gamma^+} - \frac{\partial}{\partial z}u_1\Big|_{\Gamma^-} = Q_1\frac{\partial^2}{\partial z^2}u_1\Big|_{\Gamma^-} + Q_2\frac{\partial^2}{\partial z^2}u_2\Big|_{\Gamma^+}, \tag{3}$$

where

$$Q_1 = \frac{b_{11} - 1}{i\omega\varepsilon_0 b_{12}}, \quad Q_2 = \frac{b_{22} - 1}{i\omega\varepsilon_0 b_{12}},$$

i is the imaginary unity, V is the preset potential difference between the outer planes of disk D, and the condition at infinity,

$$u_j'(M) \longrightarrow 0, \quad M \longrightarrow \infty, \quad j = 1, 2.$$

In the formulation of problem (1)–(3), the disk is treated as a capacitor with electric charges of opposite polarities (varying with frequency ω) distributed on the surfaces; potential difference V is assumed to be known. With such a formulation of the problem, the disk is also a source of the electric field.

Boundary condition (3) that simulates the penetration of the electric field through screen D follows from the boundary condition [10]

$$\frac{\partial}{\partial z}u_1\Big|_{\Gamma^-} - \frac{\partial}{\partial z}u_2\Big|_{\Gamma^+} = F(Q_1 u_1|_{\Gamma^-} + Q_2 u_2|_{\Gamma^+}),$$

where the harmonic function of u satisfies the condition

$$F(u) = (\mathbf{n}, \mathrm{curl}[\mathbf{n}, \mathrm{grad}u]) = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) = -\frac{\partial^2 u}{\partial z^2},$$

$$\mathbf{n} = \mathbf{e}_z.$$

Quantities b_{mj} ($m, j = 1, 2$) are elements of 2D square matrix B given by

$$B = A_n A_{n-1} \dots A_2 A_1 = \begin{pmatrix} b_{11} & B_{12} \\ b_{21} & b_{22} \end{pmatrix},$$

where

$$A_s = \begin{pmatrix} a_{11}^{(s)} & a_{12}^{(s)} \\ a_{21}^{(s)} & a_{22}^{(s)} \end{pmatrix}, \quad a_{12}^{(s)} = i\omega\mu_s\frac{\sin(k_s\Delta_s)}{k_s},$$

$$a_{21}^{(s)} = ik_s\frac{\sin(k_s\Delta_s)}{\omega\mu_s},$$

$a_{11}^{(s)} = \cos(k_s\Delta_s)$, $k_s = \omega\sqrt{\varepsilon_s\mu_s}$, $0 \le \arg k_s < \pi$, and $s = 1, \dots, n$.

Matrix A_s connects the tangential components of the electromagnetic field on both sides of layer S_s:

$$\begin{pmatrix} \mathbf{E}_\tau^{(s+1)} \\ \mathbf{H}_\nu^{(s+1)} \end{pmatrix} = A_s\begin{pmatrix} \mathbf{E}_\tau^{(s)} \\ \mathbf{H}_\nu^{(s)} \end{pmatrix}, \quad \mathbf{H}_\nu = [\mathbf{H}, \mathbf{n}],$$

$$s = 1, \dots, n,$$

with the help of the iterative procedure, we obtain the relation [10]

$$\begin{pmatrix} \mathbf{E}_\tau^{(n)} \\ \mathbf{H}_\nu^{(n)} \end{pmatrix} = B\begin{pmatrix} \mathbf{E}_\tau^{(1)} \\ \mathbf{H}_\nu^{(1)} \end{pmatrix},$$

which is used in deriving boundary condition (3).

Taking into account the representation $u_j = u_0 + u_j'$, we will use instead of boundary conditions (1)–(3) the boundary conditions of the form

$$(u_1' - u_2')|_{\Gamma_0} = 0, \quad \frac{\partial}{\partial z}(u_1' - u_2')|_{\Gamma_0} = 0, \tag{4}$$

$$u_2'|_{\Gamma_a} + u_0|_{\Gamma^+} = u_1'|_{\Gamma_a} + u_0|_{\Gamma^-} + V, \tag{5}$$

$$\begin{aligned} &\frac{\partial}{\partial z}u_2'\Big|_{\Gamma_a} + \frac{\partial}{\partial z}u_0\Big|_{\Gamma^+} - \frac{\partial}{\partial z}u_1'\Big|_{\Gamma_a} - \frac{\partial}{\partial z}u_0\Big|_{\Gamma^-} \\ &= Q_1\frac{\partial^2}{\partial z^2}u_1\Big|_{\Gamma^-} + Q_2\frac{\partial^2}{\partial z^2}u_2\Big|_{\Gamma^+}. \end{aligned} \tag{6}$$

Problem (4)–(6) is a modification of problem (1)–(3) in which the thin disk D is replaced by a perfectly thin disk Γ_1. On edge $\gamma = \{z = 0, \rho = a, 0 \le \varphi \le 2\pi\}$ of disk Γ_z, the condition of finiteness of the electric energy in neighborhood D_γ of edge γ must be satisfied:

$$\int_{D_\gamma} |\mathrm{grad}u|^2 dV < \infty, \quad u(M) = u_j(M),$$

$$M \in D_j, \quad j = 1, 2.$$

Such a simulation technique was used in [12, 13] for calculating the fields in perfectly thin structures with an edge.

The actual electric potentials and electric fields are defined as

$$V_j = \mathrm{Re}(u_j \exp(-i\omega t)), \quad \mathbf{E}_j = -\mathrm{grad}V_j,$$

$$j = 1, 2,$$

where t is the time.

2. FULFILLMENT OF BOUNDARY CONDITIONS

We can write potentials u_j' $(j = 1, 2)$ in the form of a superposition of cylindrical solutions to the Laplace equations [4, 12, 14] so that the condition at infinity

$$u_1'(\rho, z) = \int_0^\infty x(\lambda) J_0(\lambda\rho)\exp(\lambda z) d\lambda, \quad z < 0, \tag{7}$$

$$u_2'(\rho, z) = \int_0^\infty y(\lambda) J_0(\lambda\rho)\exp(-\lambda z) d\lambda, \quad z > 0, \tag{8}$$

be satisfied, where unknown functions $x(\lambda)$ and $y(\lambda)$ should be determined from the boundary conditions and $J_0(x)$ is the Bessel function [14, 15].

The electric field potential of a dipole with moment $\mathbf{p} = p\mathbf{e}_z$ directed along the Oz axis is given by

$$u_0(M) = \mathbf{p}\frac{z+h}{(\rho^2 + (z+h)^2)^{3/2}} = \mathbf{p}\frac{1}{r_1^2}P_1(\cos\theta_1), \quad \mathbf{p} = \frac{p}{4\pi\varepsilon_0}, \tag{9}$$

where $P_n(x)$ are the Legendre polynomials [14, 15] and (r_1, θ_1) are the spherical coordinates of point M in the spherical system of coordinates with the origin at point O_1.

Using a tabulated integral [16], we can write potential u_0 in terms of the cylindrical function:

$$u_0 = \int_0^\infty A_0(\lambda) J_0(\lambda\rho)\exp(-\lambda z) d\lambda, \quad A_0(\lambda) = \frac{\lambda p}{4\pi\varepsilon_0}e^{-\lambda h}, \quad z > -h. \tag{10}$$

Taking into account representations (7), (8), and (10) and satisfying boundary conditions (4) and (5), we obtain

$$\int_0^\infty (y(\lambda) - x(\lambda)) J_0(\lambda\rho) d\lambda = 0, \quad \rho > a, \tag{11}$$

$$\int_0^\infty \lambda(x(\lambda) + y(\lambda)) J_0(\lambda\rho) d\lambda = 0, \quad \rho > a, \tag{12}$$

$$\int_0^\infty (y(\lambda) - x(\lambda)) J_0(\lambda\rho) d\lambda = V + u_0|_{z=-\Delta/2} - u_0|_{z=\Delta/2}, \quad 0 \le \rho < a. \tag{13}$$

Let us introduce new functions [16]

$$V(\rho) = \begin{cases} v, & 0 \le \rho < a, \\ 0, & \rho > a, \end{cases} = \int_0^\infty q_V(\lambda) J_0(\lambda\rho) d\lambda, \quad q_V(\lambda) = aVJ_1(\lambda a), \tag{14}$$

$$\nu_0(\rho) = \begin{cases} u_0|_{z=\Delta/2} - u_0|_{z=-\Delta/2} = \mathbf{p}H(\rho), & 0 \le \rho < a, \\ 0, & \rho > a, \end{cases} \tag{15}$$

where, in accordance with representation (9),

$$H(\rho) = u_0|_{z=\Delta/2} - u_0|_{z=-\Delta/2} = \frac{h + \Delta/2}{(\rho^2 + (h+\Delta/2)^2)^{3/2}} - \frac{h - \Delta/2}{(\rho^2 + (h-\Delta/2)^2)^{3/2}}.$$

Applying the Hankel transformation to function $\nu_0(\rho)$ (15), we obtain

$$q_0(\lambda) = \int_0^\infty \nu_0(\rho) J_0(\lambda\rho)\rho d\rho = \mathbf{p}\int_0^a H(\rho) J_0(\lambda\rho)\rho d\rho. \tag{16}$$

Applying the inverse transformation to equality (16), we obtain the representation of function $\nu_0(\rho)$ in the form of the integral

$$\nu_0(\rho) = \int_0^\infty q_0(\lambda) J_0(\lambda\rho)\lambda d\lambda. \tag{17}$$

Two equalities (11) and (13) can be combined into one taking into account representations (14) and (17):

$$\int_0^\infty (y(\lambda) - x(\lambda)) J_0(\lambda\rho) d\lambda = \int_0^\infty q(\lambda) J_0(\lambda\rho) d\lambda, \quad 0 \le \rho < \infty, \tag{18}$$

where $q(\lambda) = q_v(\lambda) - \lambda q_0(\lambda) = aVJ_1(\lambda a) - \lambda q_0(\lambda)$.

Representation (18) implies that

$$x(\lambda) = y(\lambda) - q(\lambda), \quad 0 \le \lambda < \infty. \tag{19}$$

Further, we satisfy boundary condition (6) by substituting integrals (7), (8), and (10) into Eq. (6). This gives

$$\begin{aligned} &\int_0^\infty \lambda(x(\lambda) + y(\lambda)) J_0(\lambda\rho) d\lambda + \int_0^\infty \lambda^2 (Q_1 x(\lambda) \\ &+ Q_2 y(\lambda))\exp\left(\frac{-\lambda\Delta}{2}\right) J_0(\lambda\rho) d\lambda \\ &+ 2\int_0^\infty \lambda^2 A_0(\lambda) V\left(\frac{\lambda\Delta}{2}\right) J_0(\lambda\rho) d\lambda \\ &= 2\int_0^\infty \lambda A_0(\lambda)\sinh\left(\frac{\lambda\Delta}{2}\right) J_0(\lambda\rho) d\lambda, \quad 0 \le \rho < a, \end{aligned} \tag{20}$$

where

$$C(x) = 0.5(Q_1\exp(x) + Q_2\exp(-x)).$$

Using relation (19), we can eliminate function $x(\lambda)$ from equalities (12) and (20), which gives the set of equations

$$\int_0^\infty (y(\lambda) - q(\lambda)/2) J_0(\lambda\rho)\lambda d\lambda = \Phi(\rho), \quad 0 \le \rho < a,$$
$$\int_0^\infty (y(\lambda) - q(\lambda)/2) J_0(\lambda\rho)\lambda d\lambda = 0, \quad a < \rho < \infty, \tag{21}$$

where

$$\Phi(\rho) = \int_0^\infty \Big[-\eta^2 (Oy(\eta) - 0.5Q_1 q(\eta)) \exp\Big(\frac{-\eta\Delta}{2}\Big)$$
$$+ \eta A_0(\eta)\Big(\sinh\Big(\frac{\eta\Delta}{2}\Big) - \eta C\Big(\frac{\eta\Delta}{2}\Big)\Big)\Big] J_0(\eta\rho) d\eta, \tag{22}$$
$$Q = 0.5(Q_1 + Q_2).$$

Let us introduce the notation

$$\Phi_0(\rho) = \begin{cases} \Phi(\rho), & 0 \le \rho < a, \\ 0, & a < \rho < \infty \end{cases}$$

and write Eqs. (21) as a single equation:

$$\int_0^\infty (y(\lambda) - 0.5q(\lambda)) J_0(\lambda\rho) J_0(\lambda\rho)\lambda d\lambda = \Phi_0(\rho), \tag{23}$$
$$0 \le \rho < \infty.$$

Applying to this equation the inverse Hankel transformation, we obtain

$$y(\lambda) - 0.5q(\lambda) = \int_0^\infty \Phi_0(\rho) J_0(\lambda\rho)\rho d\rho$$
$$= \int_0^a \Phi(\rho) J_0(\lambda\rho)\rho d\rho. \tag{24}$$

Let us transform this relation by substituting into it the representation of function $\Phi(\rho)$ from (22) and set, using the tabulated integral [16],

$$L(\lambda, \eta) = \int_0^2 J_0(\eta\rho) J_0(\lambda\rho)\rho d\rho$$
$$= \frac{a}{\eta^2 - \lambda^2}[\eta J_1(\eta a) J_0(\lambda a) - \eta J_1(\lambda a) J_0(\eta a)].$$

Then, relation (24) yields

$$y(\lambda) + \int_0^\infty \eta \exp\Big(\frac{-\eta\Delta}{4}\Big) L(\lambda, \eta)$$
$$\times \Big[Q\eta \exp\Big(\frac{-\eta\Delta}{4}\Big) y(\eta) - \Psi(\eta)\Big] d\eta = 0.5q(\lambda), \tag{25}$$
$$0 \le \lambda < \infty,$$

where

$$\Psi(\eta) = 0.5Q_1 q(\eta)\eta \exp\Big(\frac{-\eta\Delta}{4}\Big)$$
$$+ A_0(\eta)\Big(\frac{\eta\Delta}{4}\Big)\Big(\sinh\Big(\frac{\eta\Delta}{2}\Big) - \eta C\Big(\frac{\eta\Delta}{2}\Big)\Big).$$

Further, we multiply both sides of Eq. (25) by $Q\lambda\exp(-\lambda\Delta/4)$ and set

$$z(\lambda) = Q\lambda \exp\Big(\frac{-\lambda\Delta}{4}\Big) y(\lambda) - \Psi(\lambda),$$

$$K(\lambda, \eta) = Q\lambda\eta \exp\Big(\frac{-(\lambda+\eta)\Delta}{4}\Big) L(\lambda, \eta),$$

$$G(\lambda) = A_0(\lambda)\exp\Big(\frac{\lambda\Delta}{4}\Big)\Big(\lambda C\Big(\frac{\lambda\Delta}{2}\Big) - \sinh\Big(\frac{\lambda\Delta}{2}\Big)\Big)$$
$$+ 0.25(Q_2 - Q_1)\lambda \exp\Big(\frac{-\lambda\Delta}{4}\Big) q(\lambda).$$

As a result, we obtain from Eq. (25) the Fredholm integral equation of the second kind:

$$z(\lambda) + \int_0^\infty K(\lambda, \eta) z(\eta) d\eta = G(\lambda), \quad 0 \le \lambda < \infty. \tag{26}$$

3. EVALUATION OF THE SHIELDING COEFFICIENT

Let us find the coefficient of shielding the electric field by disk Δ. To this end, we calculate the secondary electric field at point $M_0(\rho, z)$, $z > \Delta$:

$$\mathbf{E}_2(\rho, z) = -\mathrm{grad}\, u_2(\rho, z)$$
$$= \int_0^\infty \lambda(y(\lambda) + A_0(\lambda)) J_1(\lambda\rho)\exp(-\lambda z) d\lambda \mathbf{e}_\rho$$
$$+ \int_0^\infty \lambda(y(\lambda) + A_0(\lambda)) J_0(\lambda\rho)\exp(-\lambda z) d\lambda \mathbf{e}_z.$$

The primary field in the absence of the screen is given by

$$\mathbf{E}_0(\rho, z) = -\mathrm{grad}\, u_0(\rho, z)$$
$$= \int_0^\infty \lambda A_0(\lambda) J_1(\lambda\rho)\exp(-\lambda z) d\lambda \mathbf{e}_\rho$$
$$+ \int_0^\infty \lambda A_0(\lambda) J_0(\lambda\rho)\exp(-\lambda z) d\lambda \mathbf{e}_z.$$

The coefficient of shielding of the low-frequency electric field by disk D can be determined using the formula

$$K_e(\rho, z) = \frac{|\mathbf{E}_2(\rho, z)|}{|\mathbf{E}_0(\rho, z)|}.$$

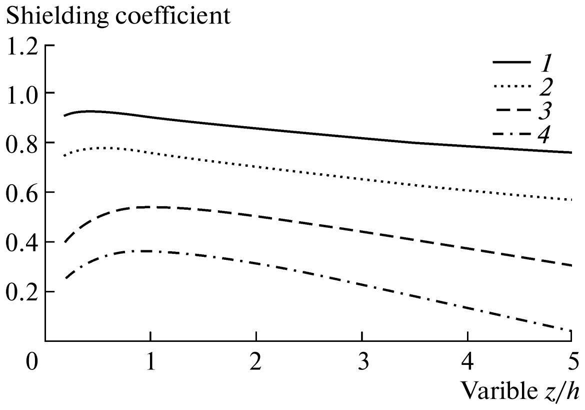


Fig. 2. Shielding coefficients $K_e(0, z)$ for some real values of ε_s, $s = 1, 2, 3$.

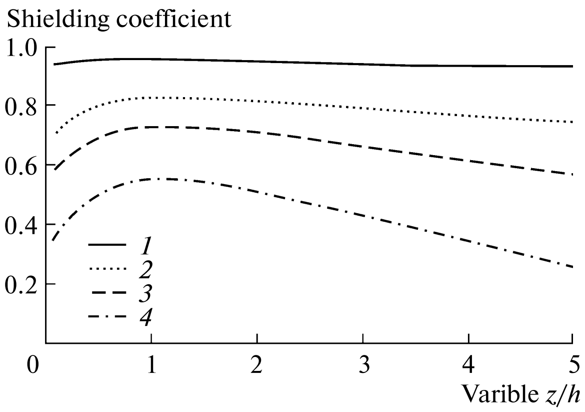


Fig. 3. Shielding coefficients $K_e(0, z)$ for some complex values of ε_s, $s = 1, 3$.

If point $M_0(\rho, z)$ lies on the z axis, then $\rho = 0$, $J_0(0) = 1$, $J_1(0) = 0$, and

$$K_e(0, z) = \left|1 + \frac{E_2'(z)}{E_0(z)}\right|, \tag{27}$$

where

$$E_2'(z) = \int_0^\infty \lambda y(\lambda)\exp(-\lambda z)d\lambda,$$

$$E_0(z) = \int_0^\infty \lambda A_0(\lambda)\exp(-\lambda z)d\lambda = 2\bar{p}(h+z)^{-3}.$$

We calculated shielding coefficient $K_e(0, z)$ using formula (27) for some parameters of the problem.

Analyzing the integral on a finite integration interval from 0 to A and using the generalized Simpson formula of the fourth order of accuracy, we transform integral Fredholm equation (26) of the second kind to the system of linear algebraic equations of the form [17]

$$z_n + \sum_{k=0}^{N} A_k K_{nk} z_k = g_n, \quad n = 0, 1, \ldots, N, \tag{29}$$

where $K_{nk} = K(\lambda_n, \lambda_k)$, $g_n = G(\lambda_n)$, $\lambda_n = nh$, $h = A/N$, N is an even number, and A_k are the weighting factors that can be calculated by the formulas

$$A_0 = A_N = \frac{h}{3}, \quad A_{2j} = \frac{2h}{3}, \quad A_{2j+1} = \frac{4h}{3},$$
$$j = 1, 2, \ldots, N/2 - 1.$$

The solution z_n, $n = 0, 1, \ldots, N$, to system (29) is assumed to be an approximate solution to integral equation (26) at points λ_n.

Calculations show that to obtain a solution to system (29) accurate to within 10^{-3} for the given parameters of the problem, we must set $A = 200$ and $h = 0.25$.

Using the solution to system (29), we can represent function $E_2'(z)$ in the form

$$\begin{aligned} E_2'(z) &= \frac{1}{Q}\int_0^\infty \psi(\lambda)\exp(-\lambda z + \lambda\Delta/4)d\lambda \\ &+ \frac{1}{Q}\sum_{k=0}^{N} A_k z_k \exp(-kh(z - \Delta/4)). \end{aligned} \tag{30}$$

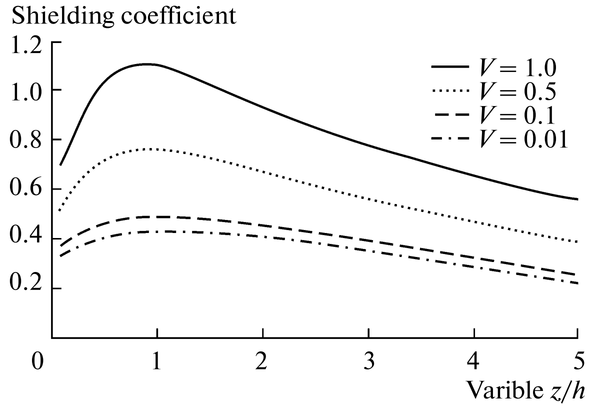
The improper integral in this expression can be evaluated to within 10^{-5} [17].

We have analyzed numerically the shielding properties of a three-layer disk with various parameters.

Figure 2 shows the $K_e(0, z)$ curves, $z > \Delta$, for the parameters $f = 1000$ Hz, $\Delta_1 = \Delta_2 = \Delta_3 = 0.05$ m, $h = 1$ m, $a = 0.7$ m, $V = 0.1$ V, $p = 1.1 \times 10^{-10}$ C m, $\mu_1 = \mu_2 = \mu_3 = \mu_0$, $\mu_0 = 4\pi \times 10^{-7}$ H/m for different values of ε: $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 4\varepsilon_0$ (curve *1*), $\varepsilon_1 = 4\varepsilon_0$, $\varepsilon_2 = 20\varepsilon_0$, $\varepsilon_3 = 50\varepsilon_0$ (*2*), $\varepsilon_1 = 50\varepsilon_0$, $\varepsilon_2 = 20\varepsilon_0$, $\varepsilon_3 = 4\varepsilon_0$ (*3*), and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 50\varepsilon_0$ (*4*), where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.

Figure 3 shows the $K_e(0, z)$ curves, $z > \Delta$, for the parameters $f = 1000$ Hz, $\Delta_1 = \Delta_2 = \Delta_3 = 0.01$ m, $h = 0.5$ m, $V = 0$, $p = 1.1 \times 10^{-10}$ C m, $\mu_1 = \mu_3 = \mu_0$, $\mu_2 = 1000\mu_0$, $\mu_0 = 4\pi \times 10^{-7}$ H/m, and $\varepsilon_2 = \varepsilon_0$ for different values of ε_1 and ε_3: $\varepsilon_1 = \varepsilon_3 = (3.5 + 2.3i)\varepsilon_0$ (*1*), $\varepsilon_1 = (3.5 + 2.3i)\varepsilon_0$, $\varepsilon_3 = (10.5 + 2.3i)\varepsilon_0$ (*2*), $\varepsilon_1 = (15.5 + 2.3i)\varepsilon_0$, $\varepsilon_3 = (3.5 + 2.3i)\varepsilon_0$ (*3*), and $\varepsilon_1 = \varepsilon_3 = (15.5 + 2.3i)\varepsilon_0$ (*4*).

Figure 4 shows the $K_e(0, z)$ curves, $z > \Delta$, for the following values of V: $V = 1$ V, $V = 0.5$ V, $V = 0.1$ V, and $V = 0.01$ V and parameters $f = 1000$ Hz, $\Delta_1 = \Delta_2 = \Delta_3 = 0.01$ m, $h = 1$ m, $a = 0.9$ m, $p = 1.1 \times 10^{-10}$ C m, $\mu_1 = \mu_3 = \mu_0$, $\mu_2 = 1000\mu_0$, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_1 = \varepsilon_3 = 100\varepsilon_0$, and $\varepsilon_2 = \varepsilon_0$.



Fig, 4. Shielding coefficients $K_e(0, z)$ for some positive values of V.

CONCLUSIONS

We have proposed a method for analytic and numerical solution of the problem of shielding of a low-frequency electric field by a multilayer disk using bilateral boundary conditions on the surface of a semitransparent thin disk.

A method has been developed for reducing the solution of the formulated boundary-value problem to the solution of the integral Fredholm equation of the second kind. We have derived an expression for calculating the coefficient of field shielding by the screen. A computer experiment has been performed. Calculations show that an increase in the permittivity of the disk reduces the coefficient of shielding (see Figs. 2 and 3), while the permeability of the layers constituting the disk weakly affects the shielding coefficient. A change in the potential difference across the outer planes of the disk affects the shielding coefficient (see Fig. 4); therefore, shielding can be controlled by varying this potential difference.

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