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SIXTH WORKSHOP ON FOURIER ANALYSIS AND RELATED FIELDS

Conference Booklet

Alfréd Rényi Institute of Mathematics,
Hungarian Academy of Sciences

Sixth Workshop on Fourier Analysis and Related Fields

Conference Booklet

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Rational Fourier – Chebyshev series of some elementary functions

Rouba, Yauheni

In the present report we consider the following system of Chebyshev – Markov rational fractions with real nonnegative parameter a:

$$M_n(x) = \cos n \arccos\left(x\frac{\sqrt{1+a^2}}{\sqrt{1+a^2x^2}}\right), \quad x \in [-1,1]\,,\; n = 0, 1, \dots .$$

It is orthogonal on the segment $[-1,1]$ with respect to the weight

$$\rho(x,a) = \frac{\sqrt{1+a^2}}{(1+a^2x^2)\sqrt{1-x^2}}, -1 < x < 1.$$

Let f be absolutely integrable with respect to the weight $\rho(x,a)$ function on the segment $[-1,1]$. We associate the Fourier series with respect to the system $\{M_n\}$ to such function f:

$$f(x) \sim \frac{c_0}{2} + \sum_{n=1}^{+\infty} c_n M_n(x),$$

with coefficients:

$$c_n = \frac{2}{\pi}\int_{-1}^{1} \rho(t,a) f(t) M_n(t)\, dt, \quad n = 0, 1, \dots .$$

Theorem 1. *For the partial sums of this Fourier series the following equality holds*

$$S_n(x;f) = \frac{1-\alpha^4}{2\pi}\int_{-\pi}^{\pi} f(\cos v)\frac{\sin{(n+1/2)\lambda(u,v)}}{\sin\lambda(u,v)/2}\frac{dv}{1+2\alpha^2\cos 2v + \alpha^4},$$

where

$$\lambda(u,v)=\int_u^v \frac{1-\alpha^4}{1+2\alpha^2\cos 2y+\alpha^4}\,dy,\quad \alpha=\frac{\sqrt{1+a^2}-1}{a}.$$

Besides, $S_n(x,f)=p_n(x)/\sqrt{(1+a^2x^2)^n}$, *where* $p_n(x)$ *is an algebraic polynomial of degree* n *and* $S_n(x,1)=1$.

In this work we study the Fourier series with respect to the system $\{M_n\}$ of the function $|x|$. We found Fourier coefficients in the explicit form. Let's introduce the following deviations:

$$\varepsilon_{2n}(x,\alpha)=|x|-s_{2n}(|x|;x),\ x\in[-1,1];\quad \varepsilon_{2n}(\alpha)=\|\varepsilon_{2n}(x,\alpha)\|_{C[-1,1]}.$$

Theorem 2. *The following inequalities hold:*

$$|\varepsilon_{2n}(x,\alpha)|\le\frac{2}{\pi}\int_0^1\sqrt{\frac{1+2\alpha^2\cos 2u+\alpha^4}{1+2t^2\cos 2u+t^4}}\,\frac{1-t^2}{1-\alpha^2t^2}\,|\chi_{2n}^*(t)|\,dt,\, x=\cos u,\qquad (*)$$

$$\varepsilon_{2n}(\alpha)\le\frac{4}{\pi}\int_0^1\left|\frac{t^2-\alpha^2}{1-\alpha^2t^2}\right|^n\frac{dt}{1+t^2}.$$

Inequality (*) *is exact in the sense that if all the poles have even multiplicity then inequality* (*) *becomes equality for* $x=0$ *and* $x=1$.

Theorem 3. *If* $\varepsilon_{2n}=\inf_\alpha\varepsilon_{2n}(\alpha)$, *then the following estimates hold*

$$\overline{\lim_{n\to\infty}}\frac{n^2}{\ln n}\varepsilon_{2n}=\frac{1}{\pi},$$

$$\inf_\alpha|\varepsilon_{2n}(x,\alpha)|\le\frac{2}{\pi\,|x|}\frac{\ln n}{n^3},\ x\in[-1,0)\cup(0,1],\ n>n_0.$$

Also, we are going to briefly discuss the other results on asymptotic estimates of elementary functions by trigonometric series.